NBEO Part I Formula Sheet: Optics and Vision Science

VERGENCE AND REFRACTION

Ametropia · 29–39 of 350 scored items

Vergence and the vergence equation GVO 3

$$L = \frac{n}{l} \qquad L' = L + F \qquad L' = \frac{n'}{l'}$$

L, L' = object and image vergence (D); l, l' = object and image distance (m); n, n' = index of the object and image media; F = power of the lens or surface (D)

Index of refraction GVO 1

$$n = \frac{c}{v}$$

n = index of refraction, which varies with wavelength; c = speed of light in a vacuum; v = speed of light in the medium

Snell's law GVO 1

$$n \sin\theta = n' \sin\theta'$$

n, n' = index of the first and second medium; θ, θ' = angles of incidence and refraction, from the normal

Critical angle GPVO 11

$$\sin\theta_c = \frac{n'}{n} \qquad (n > n')$$

θ_c = critical angle of incidence; n = index of the denser medium the light starts in; n' = index of the second medium

Beyond θ_c the light totally internally reflects.

Power of a spherical surface GVO 2

$$F = \frac{n' - n}{r}$$

F = surface power (D); n, n' = index before and after the surface; r = radius of curvature (m), positive when the center of curvature lies to the right

Thin lens in air GVO 4 · MOO 4

$$F = F_1 + F_2 = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

F_1, F_2 = front and back surface powers (D); n = lens index; r_1, r_2 = front and back radii (m), signed, so a convex back surface has $r_2 < 0$

This nominal power ignores thickness.

Apparent depth GPVO 8

$$\text{apparent distance} = \text{real distance} \times \frac{n_2}{n_1}$$

n_1 = index of the medium holding the object; n_2 = index of the observer's medium

From air, apparent depth is real depth divided by n.

Lateral magnification GVO 4

$$m = \frac{L}{L'} \qquad m = \frac{l'}{l}$$

m = image size over object size; L, L' = object and image vergence (D); l, l' = object and image distance

The l'/l form needs one medium on both sides. For a mirror, $m = -l'/l$.

Mirror power GVO 16

$$F = \frac{-2n}{r} \qquad F = \frac{-1}{f'} \qquad r = 2f'$$

F = mirror power (D); n = index of the medium in front of the mirror, 1 in air; r = radius of curvature (m); f' = focal length (m)

A concave mirror has $r < 0$, so $F > 0$. Reflected light travels right to left, so $L' = -1/l'$.

THICK LENSES AND EFFECTIVE POWER

Equivalent power GVO 6 · GPVO 16

$$F_e = F_1 + F_2 - \frac{t}{n} F_1 F_2$$

F_e = equivalent power (D); F_1, F_2 = front and back surface powers (D); t = center thickness (m); n = lens index

Two thin lenses d meters apart in air give $F_e = F_1 + F_2 - dF_1F_2$.

Back vertex power GVO 6

$$F_v = \frac{F_1}{1 - \frac{t}{n}F_1} + F_2$$

F_v = back vertex power (D); F_1, F_2 = front and back surface powers (D); t = center thickness (m); n = lens index

Front vertex (neutralizing) power swaps the surfaces, $F_n = F_2/(1 - (t/n)F_2) + F_1$.

Effective power at a new position GPVO 6

$$F_x = \frac{F}{1 - dF}$$

F = lens power at its original position (D); F_x = power that does the same job at the new position (D); d = distance moved (m), positive toward the eye

ASTIGMATISM

Transposition GPVO 10

$$S \,/\, C \times \theta \implies (S + C) \,/\, -C \times (\theta \pm 90)$$

S = sphere (D); C = cylinder (D); θ = cylinder axis (degrees)

Add 90 to an axis of 90 or less, and subtract 90 from an axis above 90.

Spherical equivalent GPVO 10

$$SE = S + \frac{C}{2}$$

SE = spherical equivalent, the dioptric position of the circle of least confusion (D); S = sphere (D); C = cylinder (D)

Power in an oblique meridian IOO 5

$$F_\theta = S + C \sin^2\theta$$

F_θ = approximate power in the meridian θ (D); S = sphere (D); C = cylinder (D); θ = angle from the cylinder axis

Javal's rule GVO app. C

$$\text{ocular astig.} = 1.25 \times \text{corneal astig.} + (-0.50 \times 090)$$

corneal astig. = keratometric astigmatism, written as minus cylinder; **ocular astig.** = predicted refractive astigmatism; -0.50×090 = allowance for against-the-rule lenticular astigmatism

THE EYE

Far point GPVO 6

$$\text{far point (m)} = \frac{1}{\text{refractive error at the cornea (D)}}$$

refractive error = correcting power at the cornea, signed (D)

A myope's far point lies in front of the eye. A hyperope's is virtual and lies behind it.

Blur and defocus GPVO 19

$$\delta = \frac{b}{d_m} V \qquad \text{angular blur (rad)} = d_m\,\delta$$

δ = defocus (D); b = blur circle diameter on the retina (m); d_m = pupil diameter (m); V = power of the eye, about 60 D

Retinal image size GPVO 6

$$|I| = 16.67\text{ mm} \times \frac{|O|}{u}$$

I = retinal image size; O = object size; u = object distance, in the units of O; 16.67 mm = nodal point to retina in a $+60.00$ D reduced eye

PHYSICAL OPTICS

Waves in a medium GPVO 21

$$v = f\lambda = \frac{c}{n} \qquad \lambda_m = \frac{\lambda}{n}$$

v = speed in the medium; f = frequency (Hz); λ = wavelength in a vacuum; λ_m = wavelength in the medium; c = speed of light in a vacuum; n = index of the medium

Frequency does not change between media.

Photon energy VP 2 · GPVO 24

$$E = h\nu = \frac{hc}{\lambda} \qquad E\,(\text{eV}) = \frac{1239}{\lambda\,(\text{nm})}$$

E = photon energy; h = Planck's constant, 6.63×10^{-34} J·s; ν = frequency (Hz); c = 3.00×10^8 m/s; λ = wavelength

Fresnel reflection GPVO 14

$$R = \left(\frac{n' - n}{n' + n}\right)^2$$

R = fraction reflected at one surface, at normal incidence; n, n' = index on each side of the surface

Quarter-wave antireflection coating GPVO 21

$$n_{\text{film}} = \sqrt{n_{\text{surround}}\, n_{\text{lens}}} \qquad t = \frac{\lambda}{4\, n_{\text{film}}}$$

n_{film} = coating index; n_{surround} = index outside the coating, 1 in air; n_{lens} = lens index; t = coating thickness for light at normal incidence; λ = design wavelength in a vacuum

Rayleigh criterion GPVO 22

$$\theta_{\min} = \frac{1.22\,\lambda}{d}$$

$\theta_{\min}$ = smallest resolvable angle (rad); λ = wavelength; d = diameter of the circular aperture, in the units of λ

Double slit and grating GPVO 21 · GPVO 22

$$x = \frac{m\lambda t}{h} \qquad h\sin\theta = m\lambda$$

x = distance of bright fringe m from the center; m = fringe order (0, 1, 2, ...); λ = wavelength; t = slit to screen distance; h = slit or grating line spacing; θ = angle to fringe m

The x form needs small angles.

Rayleigh scattering GPVO 23

$$I_{\text{scat}} \propto \frac{1}{\lambda^4}$$

I_{scat} = light scattered by particles much smaller than the wavelength; λ = wavelength

PHOTOMETRY

Illuminance from a point source VP 4

$$E = \frac{I\cos\theta}{d^2}$$

E = illuminance (lux); I = luminous intensity (cd); d = distance from the source (m); θ = angle between the surface normal and the line to the source

Luminous flux VP 4

$$\text{lumens} = 683 \times V(\lambda) \times \text{watts}$$

$V(\lambda)$ = photopic luminous efficiency, 1 at 555 nm; **watts** = radiant power at that wavelength

For mixed light, sum wavelength by wavelength.

Retinal illuminance VP 4 · EYE 5

$$\text{trolands} = L \times A$$

L = luminance (cd/m²); A = pupil area (mm²)

LENS FORM AND THICKNESS

Ophthalmic Optics / Spectacles · 17–25 items

Sagitta GVO 11 · SOD 13

$$s = r - \sqrt{r^2 - y^2} \approx \frac{y^2}{2r}$$

s = sag; r = radius of curvature; y = half the chord

Use one unit throughout. The approximation needs y much smaller than r.

Center and edge thickness GVO 11

$$ct = et + s_1 - s_2$$

ct, et = center and edge thickness; s_1, s_2 = front and back sags over the same chord, unsigned

This applies to a meniscus lens.

Lens clock reading SOD 13

$$F_{\text{surface}} = F_{\text{clock}} \times \frac{n - 1}{0.53}$$

F_{surface} = true surface power (D); F_{clock} = lens clock reading (D); n = actual lens index; $0.53 = n - 1$ for the index 1.53 the clock assumes

Spectacle magnification GPVO 18

$$SM = \frac{1}{1 - \frac{t}{n}F_1} \times \frac{1}{1 - hF_v}$$

SM = spectacle magnification; F_1 = front surface power (D); t = center thickness (m); n = lens index; F_v = back vertex power (D); h = vertex distance plus 3 mm, lens to entrance pupil (m)

The first factor is the shape factor and the second is the power factor.

PRISM

Prism diopter $P = 100 \tan d = \dfrac{\text{displacement (cm)}}{\text{distance (m)}}$ GVO 10

P = prism power (Δ); d = angle of deviation

Deviation by a thin prism $d = a(n-1)$ GVO 10 · SOD 15

d = minimum angle of deviation; a = apex angle, under about 10°, in the units of d; n = prism index

Prentice's rule $\Delta = cF$ GVO 10

Δ = induced prism (prism diopters); c = distance from the optical center (cm); F = lens power in that meridian (D)

Decentration for prism $c = \dfrac{\Delta}{F}$ SOD 16

c = decentration (cm); Δ = prism wanted (prism diopters); F = lens power in that meridian (D)

Decenter a plus lens toward the base you want and a minus lens away from it.

Resolving and combining prism $H = P\cos\theta \quad V = P\sin\theta \quad R = \sqrt{H^2+V^2}$ SOD 15 · IOO 8

$\tan\theta = \dfrac{V}{H}$

P = oblique prism to resolve (Δ); θ = base direction, 360° notation; H, V = horizontal and vertical components (Δ); R = prism that combines H and V

Find $\tan^{-1}(V/H)$ without signs, then set the quadrant: I θ, II $180-\theta$, III $180+\theta$, IV $360-\theta$.

Prism thickness difference $g = \dfrac{\Delta \cdot d}{100\,(n-1)}$ IOO 8

g = thickness difference, base to apex; Δ = prism power (prism diopters); d = lens size along the base-apex line, and g takes its units; n = lens index

MULTIFOCALS AND MATERIALS

Vertical imbalance at the reading level $VI = c\,(F_{R,90} - F_{L,90}) \qquad F_{90} \approx S + C\cos^2(\text{axis})$ SOD 21

VI = vertical imbalance (Δ); c = drop from the distance optical centers to the reading level (cm); F_{90} = power in the 90° meridian, signed (D); $F_{R,90}, F_{L,90}$ = that power for the right and left lens; S, C, axis = each lens's prescription

Image jump $\text{jump}\ (\Delta) = c \times \text{add}$ SOD 19

c = seg top to seg optical center (cm), half the diameter for a round seg; **add** = add power (D)

Distance power does not change jump.

Abbe value and lateral color $\nu_d = \dfrac{n_d - 1}{n_F - n_C} \qquad \text{TCA}\ (\Delta) = \dfrac{cF}{\nu_d}$ MOO 1

ν_d = Abbe value; n_d, n_F, n_C = index at 587.6, 486.1 and 656.3 nm; **TCA** = transverse chromatic aberration (Δ); c = distance from the optical center (cm); F = lens power (D)

Malus's law $I = I_0 \cos^2\theta$ SOD 22 · GPVO 23

I = transmitted intensity; I_0 = intensity of the already polarized incident light; θ = angle between its polarization and the transmission axis

An ideal polarizer passes 50% of unpolarized light.

Brewster's angle $\tan\theta_B = \dfrac{n'}{n}$ SOD 22

θ_B = angle of incidence at which the reflected light is fully polarized; n = index of the medium the light comes from; n' = index of the reflecting medium

ACCOMMODATION AND VERGENCE

Accommodation / Vergence / Oculomotor Function · 10–18 items

Amplitude of accommodation $A = U_{fp} - U_{np}$ GPVO 6

A = amplitude of accommodation (D); U_{fp}, U_{np} = far point and near point vergences at the cornea (D)

Hofstetter's formulas $\min = 15 - 0.25\,\text{age} \qquad \text{avg} = 18.5 - 0.30\,\text{age}$ CPOE 2

$\max = 25 - 0.40\,\text{age}$

age = age in years; $\min, \text{avg}, \max$ = expected minimum, average and maximum amplitude (D)

Tentative add $\text{add} = \dfrac{1}{\text{working distance (m)}} - \dfrac{\text{amplitude}}{2}$ CPOE 3

working distance = near working distance (m), unsigned; **amplitude** = push-up amplitude (D)

Refine the add by balancing NRA and PRA.

Meter angle $\text{MA} = \dfrac{1}{d\ (\text{m})} \qquad \Delta = \text{MA} \times \text{PD (cm)}$ CMBV 23

MA = meter angles; d = fixation distance (m); **PD** = interpupillary distance (cm); Δ = convergence demand (prism diopters)

Calculated AC/A $\text{AC/A} = \text{PD (cm)} + \text{NFD (m)} \times (H_n - H_f)$ CMBV 1

AC/A = accommodative convergence per diopter of accommodation (Δ/D); **PD** = interpupillary distance (cm); **NFD** = near fixation distance (m); H_n, H_f = near and far phorias (Δ), eso plus and exo minus

Gradient AC/A $\text{AC/A} = \dfrac{\text{phoria without lens} - \text{phoria through lens}}{\text{lens power}}$ CMBV 1

AC/A = accommodative convergence per diopter of accommodation (Δ/D); **phoria** = in Δ, eso plus and exo minus; **lens power** = the added lens, signed (D)

VISUAL PERCEPTION

Perceptual Function / Color Vision · 10–18 items

Acuity $\text{MAR (arcmin)} = \dfrac{\text{Snellen denominator}}{\text{numerator}}$ VP 7

$\log\text{MAR} = \log_{10}\text{MAR}$

MAR = minimum angle of resolution (arcmin)

A 20/20 letter subtends 5 arcmin, and its detail subtends 1 arcmin.

Contrast $C_{\text{Michelson}} = \dfrac{L_{\max} - L_{\min}}{L_{\max} + L_{\min}} \qquad C_{\text{Weber}} = \dfrac{\Delta L}{L_b}$ EYE 5 · VP 7

$L_{\max}, L_{\min}$ = peak and trough luminance; ΔL = target luminance minus background luminance; L_b = background luminance

Use Michelson contrast for gratings and Weber contrast for a target on a uniform background.

Weber and De Vries–Rose laws $\dfrac{\Delta I}{I} = K \qquad \Delta I \propto \sqrt{I}$ VP 3

ΔI = threshold increment; I = background intensity; K = Weber fraction, about 0.015 photopic and 0.14 scotopic

De Vries–Rose holds on very dim backgrounds and Weber on brighter ones.

Ricco's law $I \times A = K$ VP 3

I = threshold intensity; A = stimulus area; K = a constant

It holds inside the critical diameter, about 10 arcmin scotopic and smaller photopic.

Bloch's law $I \times t = K$ VP 3

I = threshold intensity; t = stimulus duration; K = a constant

It holds inside the critical duration, about 100 ms scotopic and 10 to 50 ms photopic.

Ferry–Porter law $\text{CFF} = k \log L + b$ ADLER 37

CFF = critical flicker fusion frequency (Hz); L = luminance; k, b = constants

It holds only under photopic conditions, and rod flicker fusion stays near 20 Hz.

CIE chromaticity coordinates $x = \dfrac{X}{X+Y+Z} \qquad y = \dfrac{Y}{X+Y+Z} \qquad x + y + z = 1$ VP 5

X, Y, Z = tristimulus values; x, y, z = chromaticity coordinates

The chromaticity diagram plots x against y.

STEREOPSIS

Amblyopia / Strabismus · 9–17 items

Stereoacuity $\eta = \dfrac{a\,\Delta b}{d^2}$ ADLER 36

η = stereo threshold (rad), times 206,265 for arcsec; a = interpupillary distance; Δb = depth interval; d = viewing distance, in the units of a and Δb

CONTACT LENSES

Contact Lenses · 8–14 items

Keratometer conversion $K\ (\text{D}) = \dfrac{337.5}{r\ (\text{mm})}$ GPVO 14

K = corneal power (D); r = anterior corneal radius (mm)

337.5 comes from the keratometric index, $(1.3375 - 1) \times 1000$.

Tear lens $\text{tear lens (D)} = \text{base curve (D)} - \text{flat K (D)}$ CLOP 7 · MOO 7

base curve = back optic zone radius, as power (D); **flat K** = flattest keratometry reading (D)

Steeper than K adds minus and flatter adds plus (SAM–FAP), about 0.25 D per 0.05 mm.

Residual astigmatism $\text{RA} = \text{refractive astigmatism} - \text{corneal astigmatism}$ CLOP 11

RA = residual astigmatism, what a spherical rigid lens leaves uncorrected; **refractive astigmatism** = cylinder in the refraction; **corneal astigmatism** = keratometric cylinder

Plain subtraction works when the axes match.

IOL POWER

Lens / Cataract / IOL · 7–13 items

SRK formula $P = A - 2.5L - 0.9K$ AOOT 6

P = IOL power for emmetropia (D); A = lens constant; L = axial length (mm); K = average keratometry (D)

LOW VISION

Low Vision · 5–9 items

Magnifier $M = \dfrac{F}{4}$ GVO 13

M = effective magnification (×); F = magnifier power (D)

It assumes a 25 cm reference distance, with the object at the focal point.

Relative distance magnification $M = \dfrac{u_1}{u_2}$ GPVO 17

M = relative distance magnification; u_1 = original viewing distance; u_2 = new viewing distance

Telescope $M = -\dfrac{F_e}{F_o} \qquad d = \dfrac{1}{F_o} + \dfrac{1}{F_e}$ GVO 14

M = angular magnification; F_o, F_e = objective and eyepiece powers (D); d = tube length (m)

A Galilean ($F_e < 0$) gives $M > 0$ and an erect image.

Kestenbaum add $\text{starting add (D)} = \dfrac{\text{Snellen denominator}}{\text{numerator}}$ AOA table 6

numerator, denominator = of the best-corrected distance Snellen acuity

For standard print, 20/200 gives $+10.00$ D.

SOURCES

GVO *Geometrical and Visual Optics: A Clinical Introduction, 3rd ed*, App. C · **GPVO** *Geometric, Physical, and Visual Optics* · **MOO** *Modern Ophthalmic Optics* · **IOO** *Introduction to Ophthalmic Optics* · **VP** *Visual Perception: A Clinical Orientation* · **EYE** *The Eye: Basic Sciences in Practice, 5th ed* · **SOD** *System for Ophthalmic Dispensing* · **CPOE** *Clinical Procedures for Ocular Examination* · **CMBV** *Clinical Management of Binocular Vision, 5th ed* · **ADLER** *Adler's Physiology of the Eye* · **CLOP** *Contact Lenses in Ophthalmic Practice* · **AOOT** *Advances in Ophthalmic Optics Technology* · **AOA** *Optometric Clinical Practice Guideline: Care of the Patient with Visual Impairment (Low Vision Rehabilitation)*, Table 6

Each tag names the book and chapter. Chapter titles are at eyeknowod.com/formula-sheet. EyeKnow O.D. is not affiliated with the NBEO.